British Mathematical Olympiad

Round 2 : Thursday, 24 February 1994

Time allowed *Three and a half hours.*

Each question is worth 10 marks.

Instructions

- *Full written solutions - not just answers - are required, with complete proofs of any assertions you may make. Marks awarded will depend on the clarity of your mathematical presentation. Work in rough first, and then draft your final version carefully before writing up your best attempt.*

  Rough work should *be handed in, but should be clearly marked.*
- *One or two* complete *solutions will gain far more credit than trying all four problems.*
- *The use of rulers and compasses is allowed, but calculators and protractors are forbidden.*
- *Staple all the pages neatly together in the top* left *hand corner, with questions 1,2,3,4 in order, and the cover sheet at the front.*

In early March, twenty students will be invited to attend the training session to be held at Trinity College, Cambridge (on 7-10 April). On the final morning of the training session, students sit a paper with just 3 Olympiad-style problems. The UK Team - six members plus one reserve - for this summer's International Mathematical Olympiad (to be held in Hong Kong, 8-20 July) will be chosen immediately thereafter. Those selected will be expected to participate in further correspondence work between April and July, and to attend a short residential session before leaving for Hong Kong.

Do not turn over until **told to do so.**

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1. Find the first integer $n > 1$ such that the average of
$$1^2, 2^2, 3^2, \ldots, n^2$$
is itself a perfect square.

2. How many different (i.e. pairwise non-congruent) triangles are there with integer sides and with perimeter 1994?

3. AP, AQ, AR, AS are chords of a given circle with the property that
$$\angle PAQ = \angle QAR = \angle RAS.$$
Prove that
$$AR(AP + AR) = AQ(AQ + AS).$$

4. How many perfect squares are there (mod 2^n)?